

P3 30% of A Levels
75 MARKS
1h 50min
11 Questions

DIFFICULTY ↑.

Binomial
Partial
Polynomials
Logs
Trig
DIFF
integ
Diff eq

P3 BINOMIAL

$(a+b)^n = a^n + {}^n C_1 a^{n-1} b^1 + {}^n C_2 a^{n-2} b^2 \dots$
n is integer > 1, {n=2, 3, 4, 5, ...}

P3 BINOMIAL

must be 1
 $(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \dots$
when n is negative or fraction/decimal

CONDITION $-1 < x < 1$

FACTORIAL: $3! = 3 \times 2 \times 1$ $x!$

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$1! = 1$, $0! = 1$

Q: Expand first four terms in expansion of

(i) $(1+2x)^{-1}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!}$$

$$= 1 + (-1)(2x) + \frac{(-1)(-1-1)(2x)^2}{2!} + \frac{(-1)(-1-1)(-1-2)(2x)^3}{3!}$$

$$\boxed{(-1)(-1-1)(2)^2 \div 2! = 4}$$

$$\boxed{(-1)(-1-1)(-1-2)(2)^3 \div 3! = -8}$$

$$= 1 - 2x + 4x^2 - 8x^3$$

2 $(1-2x)^{\frac{1}{2}}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \dots$$

$$1 + \left(\frac{1}{2}\right)(-2x) + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\frac{(-2x)^2}{2!} + \left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\frac{(-2x)^3}{3!}$$

$$\boxed{(1 \div 2)(1 \div 2 - 1)(-2)^2 \div 2! = -\frac{1}{2}}$$

$$\boxed{(1 \div 2)(1 \div 2 - 1)(1 \div 2 - 2)(-2)^3 \div 3! = -\frac{1}{2}}$$

$$1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3$$

3 $\left(1 - \frac{2}{3}x\right)^{\frac{3}{2}}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \dots$$

$$= 1 + \left(\frac{3}{2}\right)(-\frac{2}{3}x) + \left(\frac{3}{2}\right)\left(\frac{3}{2}-1\right)\left(-\frac{2}{3}x\right)^2$$

$$(2)(3) / \frac{(2)(2)(3)}{2!}$$

$$(3 \div 2)(3 \div 2 - 1)(-2 \div 3)^2 \div 2! = \frac{1}{6}$$

$$= 1 - x + \frac{1}{6}x^2$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} \dots$$

↓

IF THIS IS NOT 1....

$$(3x + 2)^{-1}$$

Correct form: (constant + variable)ⁿ

$$(2 + 3x)^{-1}$$

~~$$2 \left(1 + \frac{3x}{2} \right)^{-1}$$~~

→ MOST IMPORTANT BLUNDER TO REMEMBER.

$$\left[2 \left(1 + \frac{3x}{2} \right) \right]^{-1}$$

$$2^{-1} \left(1 + \frac{3x}{2} \right)^{-1}$$

$$\frac{1}{2} \left[1 + (-1) \left(\frac{3x}{2} \right) + \frac{(-1)(-1-1)}{2!} \left(\frac{3x}{2} \right)^2 \right]$$

$$\frac{1}{2} \left(1 - \frac{3}{2}x + \frac{9}{4}x^2 \right)$$

2) $(x+2)^{-2}$ Expand first three terms.

$$(2+x)^{-2}$$
$$\left[2 \left(\frac{1+x}{2}\right)\right]^{-2}$$
$$2^{-2} \left(\frac{1+x}{2}\right)^{-2}$$

$$\frac{1}{4} \left[1 + (-2) \left(\frac{x}{2}\right) + \frac{(-2)(-2-1)}{2!} \left(\frac{x}{2}\right)^2 \right]$$

$$\frac{1}{4} \left[1 - x + \frac{3}{4} x^2 \right]$$

Q: $f(x) = \frac{-1}{x-1} + \frac{4}{x-2} - \frac{2}{x+1}$

Show that if expanded upto x^3 term,

$$f(x) = -3 + 2x - \frac{3}{2} x^2 + \frac{11}{4} x^3 \quad (5 \text{ mark})$$

Solution $f(x) = \frac{-1}{x-1} + \frac{4}{x-2} - \frac{2}{x+1}$

$$\underbrace{-1}_{W1} (x-1)^{-1} + \underbrace{4}_{W2} (x-2)^{-1} - \underbrace{2}_{W3} (x+1)^{-1}$$

$$-1 \left[-1(1+x+x^2+x^3) \right] + \frac{2}{4} \left[-\frac{1}{2} \left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} \right) \right] - 2 \left[1 - x + x^2 - x^3 \right]$$

$$1 + x + x^2 + x^3 - 2 - x - \frac{x^2}{2} - \frac{x^3}{4} - 2 + 2x - 2x^2 + 2x^3$$

$$-3 + 2x - \frac{3}{2}x^2 + \frac{11}{4}x^3$$

$$\begin{aligned}
 & (W1) \quad (x-1)^{-1} \\
 & \quad (-1+x)^{-1} \\
 & \quad [-1(1-x)]^{-1} \\
 & \quad (-1)^{-1}(1-x)^{-1} \quad \boxed{(-1)(-1)(-1)^2 \div 2! = 1} \quad \boxed{(-1)(-1)(-1-2)(-1)^3 \div 3! = 1} \\
 & \quad -1 \left[1 + (-1)(-x) + \frac{(-1)(-1)(-x)^2}{2!} + \frac{(-1)(-1)(-1-2)(-x)^3}{3!} \right] \\
 & \quad \boxed{-1(1+x+x^2+x^3)} \rightarrow (W1)
 \end{aligned}$$

$$\begin{aligned}
 & (W2) \quad (x-2)^{-1} \\
 & \quad (-2+x)^{-1} \\
 & \quad \left[-2 \left(1 - \frac{x}{2} \right) \right]^{-1} \\
 & \quad (-2)^{-1} \left(1 - \frac{x}{2} \right)^{-1} \\
 & \quad -\frac{1}{2} \left[1 + (-1) \left(-\frac{x}{2} \right) + \frac{(-1)(-1-1) \left(-\frac{x}{2} \right)^2}{2!} + \frac{(-1)(-1-1)(-1-2) \left(-\frac{x}{2} \right)^3}{3!} \right] \\
 & \quad \boxed{-\frac{1}{2} \left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} \right)} \rightarrow (W2)
 \end{aligned}$$

$$\begin{aligned}
 & (W3) \quad (x+1)^{-1} \\
 & \quad (1+x)^{-1} \\
 & \quad 1 + (-1)(x) + \frac{(-1)(-1-1)(x)^2}{2!} + \frac{(-1)(-1-1)(-1-2)(x)^3}{3!} \\
 & \quad \boxed{1 - x + x^2 - x^3} \rightarrow (W3)
 \end{aligned}$$

